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Dynamical transmission and effect of smoking in society

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ABSTRACT

Smoking is a large problem in the entire world. Despite overwhelming facts about smoking, it is still a very bad habit which is widely spread and accepted socially. Among smokers, often the desire to quit smoking arises. A large number of smokers attempt to quit, but only a few of them are successful. In this research, the nonstandard finite difference scheme is applied on system which is dynamically consistent, easy to implement and show a good agreement to control the bad impact of smoking for long period of time and to eradicate a death killer factor in the world spread by smoking. We have discussed the qualitative behavior of the model and numerical simulations are carried out to support the analytical results.

1. Introduction

The scope of mathematics includes mathematical modeling and esoteric mathematics. The flow of work, process, predictions and outcomes can easily be measured with the help of mathematical concepts and theory. Therefore, biologists are now extremely dependent on mathematics. Mathematical modeling of biological sciences is done by many brilliant scientists (Biazar, 2006; Busenberg and Van den Driessche, 1990; El-Sayed et al., 2009). The relationship between simple mathematical modeling involves biological system, integer order differential equations that show their dynamics and complex system which describes their changing of structure. The nonlinearity and multi-scale behaviors in mathematical modeling describe the mutual relationship between parameters (Makinde, 2007). In last few decades, many biological models were studied in detail by using classical derivatives (Arafa et al., 2012; Kribs-Zaleta, 1999; Buonomo and Lacitignola, 2008; Liu and Wang, 2010; Haq et al, 2017).

Smoking is the major problem in the entire world effecting healthy community. Smoking effects different organs of human body caused more than one million deaths in the world. A chance of heart attack in smoker is 70% more as compared to nonsmoker. Similarly the incident rate of lung cancer of smoker is 10% more than nonsmoker. The main effects of short term smoking are coughing, stained teeth, high blood pressure and bad breath. The major effects of long term smoking are gum disease, stomach ulcer, lung cancer, heart disease, throat cancer and mouth cancer in the recent years. The life of smoker is also 12 to 13 years shorter than non-smoker. According to the reports of WHO smoking kills many individual in the entire world. Every scientist, doctor and mathematician tries to control the effect of smoking and mathematician tried for the formation of different valuable smoking models to overcome the smoking effects. The lot of smoking models was planned by the authors. Examined the smoking model related with Caputo fractional derivative (Erturk et al., 2012). Examined the optimal control of the smoking models and present the qualitative analysis of dynamics of smoking (Zaman, 2011a; 2011b). Analysis of cigarette smoking and lung cancer (Lubin and Caporaso, 2006). Describe the mathematical analysis of the dynamics of tobacco us their recovery and decline (Garsow et al., 2000). Explained the smoking fractional mathematical model (Mickens, 1989). Established the curtailing smoking dynamics (Sharomi and Gumel, 2008). Examined a fractional smoking and many others (Zeb et al., 2012). Interpret the description of smoking global dynamics of mathematical system of equations (Alkhudhari et al., 2014).

In order to overcome the numerical instabilities phenomena in 1989 Mickens introduced the concept nonstandard finite difference scheme and after that has developed NSFD methods in many works, such as (Mickens, 1994; 2000; 2002; 2005). According to Mickens, NSFD schemes are those constructed following a set of five basic rules. The NSFD schemes preserve main properties of the differential counterparts, such as positivity, monotonicity,

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periodicity, stability, and some other invariant including energy and geometrical shapes. It should be emphasized that NSFD schemes can preserve all properties of continuous models for any discretization parameters. The discrete models with these properties are called dynamically consistent (Anguelov et al., 2014).

In this paper, the total population is divided into five compartments potential smokers, occasional smokers, heavy smokers, temporary quitters and permanently quitters. We also investigate the stability and qualitative analysis of smoking model. An unconditionally convergent nonstandard finite difference scheme has been presented to obtain solution of model. The analysis of disease free equilibrium has been made by finding reproductive number. Numerical results are presented graphically to show the dynamics of the model.

2. Mathematical model

Smoking model is divided into five sub compartment like potential smokers $P(t)$, occasional smokers $L(t)$, heavy smokers $S(t)$ temporary quitters $Q(t)$ and smokers who quit permanently $R(t)$ specified by $T(t) = P(t) + L(t) + S(t) + Q(t) + R(t)$. The proposed smoking model (Singh et al., 2017) in the form of system of nonlinear differential equation is given as:

$$\frac{dP}{dt} = a(1-P) - bPS \qquad (1)$$
$$\frac{dL}{dt} = -aL + bPS - cLS \qquad (2)$$
$$\frac{dS}{dt} = -(a+d)S + cLS + fQ \qquad (3)$$
$$\frac{dQ}{dt} = -(a+f)Q + d(1-e)S \qquad (4)$$
$$\frac{dR}{dt} = -aR + edS \qquad (5)$$

Here b indicates the contact rate between potential smokers and smokers who smoke occasionally, c represents the contact rate between temporary quitters and smokers who smoke occasionally, d represents the rate of giving up smoking, $(1-e)$ stances for the fraction of smokers who temporarily give up smoking at a rate d, f indicate the contact rate between smokers and temporary quitters who return back to smoking, a denotes the rate of natural death, e denotes the remaining fraction of smokers who give up smoking permanently (at a rate d).

3. Qualitative analysis

3.1. Disease free equilibrium

By substituting the values of parameters in given system of differential equations and the rate of change with respect to time is zero, we get.

$$a(1-P) - bPS = 0$$
$$-aL + bPS - cLS = 0$$
$$-(a+d)S + cLS + fQ = 0$$
$$-(a+f)Q + d(1-e)S = 0$$
$$-aR + edS = 0$$

By simplifying the above equations we get, disease-free equilibrium, denoted by E_0 i.e., $E_0 = (1,0,0,0,0)$.

3.2. Endemic equilibrium

Endemic equilibrium are found in terms of one of the infected compartment, denoted by E_1 i.e.,

$$E_1 = (P^*, L^*, S^*, Q^*, R^*)$$

where

$$P^* = \frac{a}{a+bS^*}, L^* = \frac{ab}{(a+bS^*)(a+cS^*)}, Q^* = \frac{d(1-e)S^*}{a+f}, R^* = \frac{edS^*}{a}$$

4. Stability and sensitivity analysis

It is important to find the verge conditions to check the status of population, whether the disease persist or dies out. In case of disease free equilibrium point, $R_0 < 1$, which shows that the disease will dies out. In case of endemic equilibrium, $R_0 > 1$, which shows that the disease spreads in the population, where R_0 is the reproductive number is also known basic reproductive ratio or basic reproductive rate. Consider the jacobian matrix as

$$J = \begin{bmatrix} -a-bS & 0 & -bP & 0 & 0 \\ bS & -a-cS & bP-cL & 0 & 0 \\ 0 & cS & -a-d+cL & f & 0 \\ 0 & 0 & d(1-e) & -a-f & 0 \\ 0 & 0 & ed & 0 & -a \end{bmatrix}$$

since the Jacobin matrix is $J = F - V$ where

$$F = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & f & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

and

$$V = \begin{bmatrix} a+bS & 0 & bP & 0 & 0 \\ -bS & a+cS & -bP+cL & 0 & 0 \\ 0 & -cS & a+d-cL & f & 0 \\ 0 & 0 & -d(1-e) & a+f & 0 \\ 0 & 0 & -ed & 0 & a \end{bmatrix}$$

We know that $K = FV^{-1}$ and using the relation $|K - \lambda I| = 0$ solving on mathematica for the Eigen value λ, which represents the reproductive number R_0 i.e.,

$$R_0 = \frac{df(1-e)}{(a+d)(a+f)}$$

hence $R_0 = 0.431034 < 1$, according the parameters values given by Singh et al. (2017).

Theorem 1: The disease free equilibrium $E_0$is locally asymptotically stable for $R_0 < 1$, otherwise unstable.

Proof: E_0 of the given system is locally asymptotically stable if $Re(\lambda) < 0$ where λ can be evaluated from the relation $|J_0 - \lambda \mathrm{I}| = 0$.

Consider the jacobian matrix again and substituting the values of disease free point E_0, we get

$$J_0 = \begin{bmatrix} -a & 0 & -b & 0 & 0 \\ 0 & -a & b & 0 & 0 \\ 0 & 0 & -a-d & f & 0 \\ 0 & 0 & d(1-e) & -a-f & 0 \\ 0 & 0 & ed & 0 & -a \end{bmatrix}$$

By using the relation $|J_0 - \lambda I| = 0$, we get. $Re(\lambda)$ as:

$$\lambda_1 = -a,$$
$$\lambda_2 = \frac{1}{2}\left[-2a - d - f - \sqrt{d^2 + 2df - 4def + f^2}\right] < 0,$$
$$\lambda_3 = \frac{1}{2}\left[-2a - d - f + \sqrt{d^2 + 2df - 4def + f^2}\right] < 0.$$

All the Eigen values of the above matrix according to the disease free equilibrium point are negative real parts which represents that the given system is locally asymptotically stable.

Sensitivity of R_0 can be analyzed by taking the partial derivatives of reproductive number for the involved parameters as follows

$$\frac{\partial R}{\partial a} = \frac{-df(1-e)(2a+d+f)}{\left((a+d)(a+f)\right)^2} < 0$$
$$\frac{\partial R}{\partial d} = \frac{af(1-e)}{(a+f)(a+d)^2} > 0$$
$$\frac{\partial R}{\partial e} = \frac{-df}{(a+d)(a+f)} < 0$$
$$\frac{\partial R}{\partial f} = \frac{ad(1-e)}{(a+d)(a+f)^2} > 0.$$

It can be seen that R_0 is most sensitive to change in parameter, here, R_0 is increasing with d, f and decreasing with a, e . In other words it found that the sensitivity analysis shows that prevention is better than to control the disease.

5. Non standard fine difference scheme

In this section, we design an NSFD scheme (Anguelov and Lubuma, 2001) that replicates the dynamics of the continuous model (1)-(5). Let $Y_k = (P_k, L_k, S_k, Q_k, R_k)^T$ denoted an approximation of $X(t_k)$ where $t_k = k\Delta t$ with $k \in N, h = \Delta t > 0$ be a step size, then

$$\frac{P^{k+1}-P^k}{\emptyset} = a - aP^{k+1} - bP^{k+1}S^k \quad (6)$$
$$\frac{L^{k+1}-L^k}{\emptyset} = -aL^{k+1} + bP^{k+1}S^k - cL^{k+1}S^k \quad (7)$$
$$\frac{S^{k+1}-S^k}{\emptyset} = -(a+d)S^{k+1} + cL^{k+1}S^k + fQ^k \quad (8)$$
$$\frac{Q^{k+1}-Q^k}{\emptyset} = -(a+f)Q^{k+1} + d(1-e)S^{k+1} \quad (9)$$
$$\frac{R^{k+1}-R^k}{\emptyset} = -aR^{k+1} + edS^{k+1} \quad (10)$$

which is the proposed NSFD Scheme for the given model, where

$$\emptyset = \emptyset(h) = \frac{1-e^{-d(1-e)h}}{d(1-e)} \quad (11)$$

The discrete method (6-10) is indeed an NSFD scheme because it is constructed according to Mickens' rules (Mickens, 1994) formalized as follows in (Anguelov and Lubuma, 2001).

Rule 1: The standard denominator $h = \Delta t$ of the discrete derivatives is replaced by the complex denominator function in Eq. 11 which satisfies the asymptotic relation $\emptyset(h) = h + O(h^2)$.

Note that the denominator function $\emptyset$ is expected to better capture the dynamics of the continuous model through the presence of the underlying parameters d, e. In fact, exact schemes for a wide range of dynamical systems involve such complex denominator functions (Lubuma and Patidar, 2007; Gumel, 2014).

Rule 2: Nonlinear terms in the right-hand side of Eqs. 1-5 are approximated in a non-local way. For instance, $P(t_k)S(t_k) \cong P_{k+1}S_k$ we have instead of $P(t_k)S(t_k) \cong P_kS_k$

6. Analysis of scheme

Theorem 6.1: The NSFD scheme (6)-(10) is a dynamical system on the biological feasible domain κ of the continuous model (1)-(5).

Proof: First, we prove the positivity of the scheme (6-10). It is easy to show that the NSFD scheme (6-10) takes the explicit form

$$P^{k+1} = \frac{\emptyset a + P^k}{1+\emptyset(a+bS^k)}$$
$$L^{k+1} = \frac{\emptyset b(\emptyset a + P^k)S^k + \left(1+\emptyset(a+bS^k)\right)L^k}{(1+\emptyset(a+bS^k))(1+\emptyset(a+cS^k))}$$
$$S^{k+1} = \frac{\emptyset cA^*S^k + \emptyset fB^*Q^k + B^*S^k}{B^*(1+\emptyset(a+d))}$$
$$Q^{k+1} = \frac{\emptyset d(1-e)\left(\emptyset cA^*S^k + \emptyset fB^*Q^k + B^*S^k\right) + B^*(1+\emptyset(a+d))Q^k}{B^*(1+\emptyset(a+d))(1+\emptyset(a+f))}$$
$$R^{k+1} = \frac{\emptyset ed\left(\emptyset cA^*S^k + \emptyset fB^*Q^k + B^*S^k\right) + B^*(1+\emptyset(a+d))R^k}{B^*(1+\emptyset(a+d))(1+\emptyset a)}$$

where

$$A^* = \emptyset b(\emptyset a + P^k)S^k + \left(1 + \emptyset(a + bS^k)\right)L^k$$

and

$$B^* = \left(1 + \emptyset(a + bS^k)\right)\left(1 + \emptyset(a + cS^k)\right).$$

Thus $P^{k+1} \geq 0$, $L^{k+1} \geq 0$, $S^{k+1} \geq 0$, $Q^{k+1} \geq 0$, $R^{k+1} \geq 0$, whenever the discrete variables are non-negative at the previous iteration. It remains to prove the positive invariance of κ. Adding the (6), (7) and (8), we have

$$1 + \emptyset(a + d)H_{k+1} = \emptyset a + H_k - (1 + \emptyset a)I_{k+1} \leq \emptyset a + H_k$$
$$1 + \emptyset(a + d)H_{k+1} \leq \emptyset a + H_k$$

therefore $H_{k+1} \leq \frac{a}{a+d}$ whenever $H_k \leq \frac{a}{a+d}$.

The priori bounds for Q_{k+1} and R_{k+1} follows readily from the fact that $Q_{k+1}$and L_{k+1} are less than or equal to H_{k+1}. This completes the proof.

7. Results and discussion

The mathematical analysis of epidemic smoking model with non-linear incidence has been presented. To observe the effects of the parameters using in this dynamics smoking model (1)-(5), conclude several numerical simulations varying the value of parameters for $R_0 < 1$. Figs. 1-5 show the convergence solution for diseases free equilibrium by using NSFD scheme at $h = 1, h = 5$ and $h = 10$ for $\emptyset = \emptyset(h) + O(h^2)$. The technique create a better impact to control the smoking, it reduces the infected rate and increases the potential smokers during disease Free State. In Fig. 1 by decreasing the value of h potential smokers(P)increases with time. In Fig. 2 by decreasing the value of h occasional smokers (L) decreases rapidly with time. In Fig. 3 by decreasing the value of h heavy smokers (S) decreases rapidly with time. In Fig. 4, by decreasing the value of h temporary quitters (Q) increases initially, but after some time decreases rapidly with time. In Fig. 5 by decreasing the value of h permanent quitters (R) increases initially, but after some time decreases rapidly with time. It can be easily seen that by reducing the step size the system (1)-(5) converge rapidly to the steady state point.

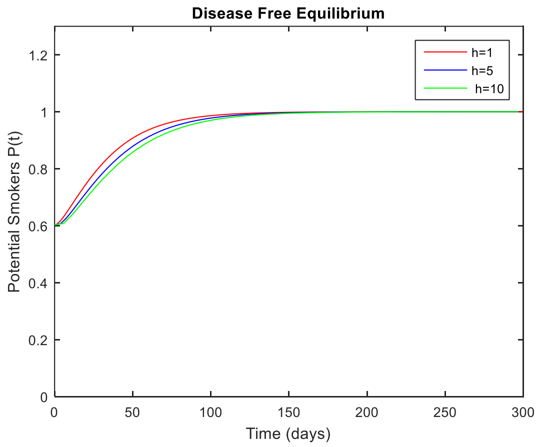

Fig. 1: Numerical solutions for potential smokers in a time t with step size $h = 1, h = 5$ and $h = 10$ for disease free equilibrium points

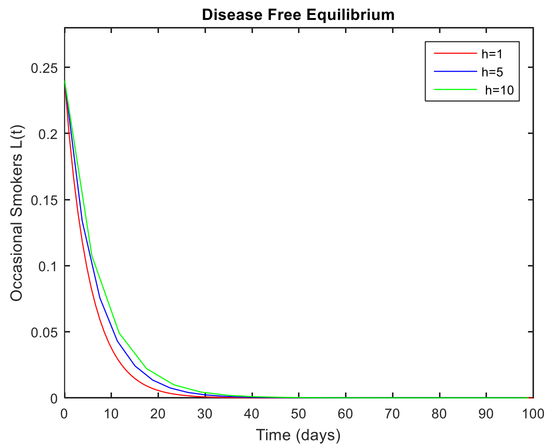

Fig. 2: Numerical solutions for occasional smokers in a time t with step size $h = 1, h = 5$ and $h = 10$ for disease free equilibrium points

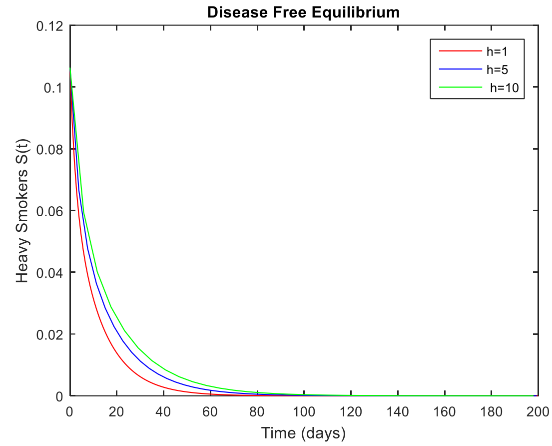

Fig. 3: Numerical solutions for heavy smokers in a time t with step size $h = 1, h = 5$ and $h = 10$ for disease free equilibrium points

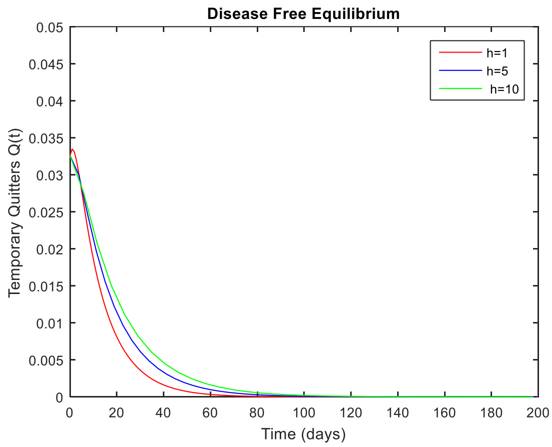

Fig. 4: Numerical solutions for temporary quitters in a time t with step size $h = 1, h = 5$ and $h = 10$ for disease free equilibrium points

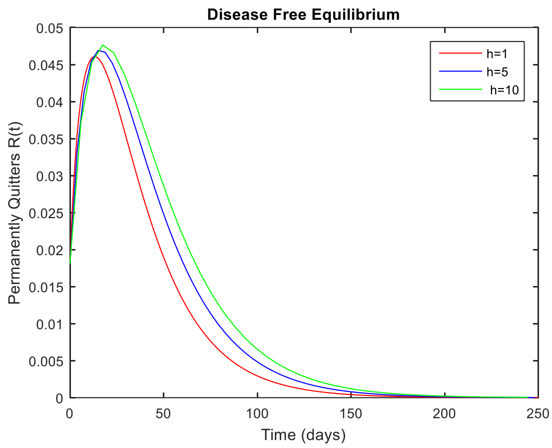

Fig. 5: Numerical solutions for permanently quitters in a time t with step size $h = 1, h = 5$ and $h = 10$ for disease free equilibrium points

8. Conclusion

Sufficient conditions for local stability of the DFE point E_0 are given by using the basic reproduction number R_0 of the model, where it is asymptotically stable and sensitivity analysis of the parameters involved in threshold parameter R_0, which shows the actual behavior of the dynamical model to reduce the smoking effect in the society. It is important to note that nonstandard finite difference scheme for mathematical models based on system of differential equations is more powerful approach to compute the convergent solutions for the disease models. Finally, we presented the numerical simulation and verified all the analytical results numerically by using nonstandard finite difference scheme to reduce the infected rates very fast for disease free equilibria by using different step size, we are able to control the spreading of smoking in the community.

References

Alkhudhari Z, Al-Sheikh S, and Al-Tuwairqi S (2014). Global dynamics of a mathematical model on smoking. Hindawi Publishing Corporation, ISRN Applied Mathematics, 2014: Article ID 847075, 7 pages. http://doi.org/10.1155/2014/847075

Anguelov R and Lubuma JMS (2001). Contributions to the mathematics of the nonstandard finite difference method and applications. Numerical Methods for Partial Differential Equations, 17(5): 518-543.

Anguelov R, Dumont Y, Lubuma JS, and Shillor M (2014). Dynamically consistent nonstandard finite difference schemes for epidemiological models. Journal of Computational and Applied Mathematics, 255: 161-182.

Arafa AAM, Rida SZ, and Khalil M (2012). Fractional modeling dynamics of HIV and CD4+ T-cells during primary infection. Nonlinear Biomedical Physics, 6(1): 1-7.

Biazar J (2006). Solution of the epidemic model by Adomian decomposition method. Applied Mathematics and Computation, 173(2): 1101-1106.

Buonomo B and Lacitignola D (2008). On the dynamics of an SEIR epidemic model with a convex incidence rate. Ricerche Di Matematica, 57(2): 261-281.

Busenberg S and Van den Driessche P (1990). Analysis of a disease transmission model in a population with varying size. Journal of Mathematical Biology, 28(3): 257-270.

El-Sayed AMA, Rida SZ, and Arafa AAM (2009). On the solutions of time-fractional bacterial chemotaxis in a diffusion gradient chamber. International Journal of Nonlinear Science, 7(4): 485-492.

Erturk VS, Zaman G, and Momani S (2012). A numeric–analytic method for approximating a giving up smoking model containing fractional derivatives. Computers and Mathematics with Applications, 64(10): 3065-3074.

Garsow CC, Salivia GJ, and Herrera AR (2000). Mathematical models for the dynamics of tobacoo use, recovery and relapse. Technical Report Series BU-1505-M, Cornell University, UK.

Gumel A (2014). Mathematics of Continuous and Discrete Dynamical Systems. Vol. 618, American Mathematical Society, Providence, Rhode Island.

Haq F, Shah K, ur Rahman G, and Shahzad M (2017). Numerical solution of fractional order smoking model via laplace Adomian decomposition method. Alexandria Engineering Journal. https://doi.org/10.1016/j.aej.2017.02.015

Kribs-Zaleta CM (1999). Structured models for heterosexual disease transmission. Mathematical Biosciences, 160(1): 83-108.

Liu X and Wang C (2010). Bifurcation of a predator prey model with disease in the prey. Nonlinear Dynamics, 62(4): 841-850.

Lubin JH and Caporaso NE (2006). Cigarette smoking and lung cancer: modeling total exposure and intensity. Cancer Epidemiology and Prevention Biomarkers, 15(3): 517-523.

Lubuma JMS and Patidar KC (2007). Non-standard methods for singularly perturbed problems possessing oscillatory/layer solutions. Applied Mathematics and Computation, 187(2): 1147-1160.

Makinde OD (2007). Adomian decomposition approach to a SIR epidemic model with constant vaccination strategy. Applvcied Mathematics and Computation, 184(2): 842-848.

Mickens RE (1989). Exact solutions to a finite-difference model of a nonlinear reaction-advection equation: Implications for numerical analysis. Numerical Methods for Partial Differential Equations, 5(4): 313-325.

Mickens RE (1994). Nonstandard finite difference models of differential equations. World Scientific, Singapore, Singapore.

Mickens RE (2000). Applications of nonstandard finite difference schemes. World Scientific, Singapore, Singapore.

Mickens RE (2002). Nonstandard finite difference schemes for differential equations. The Journal of Difference Equations and Applications, 8(9): 823-847.

Mickens RE (2005). Advances in the applications of nonstandard finite difference schemes. World Scientific, Singapore, Singapore.

Sharomi O and Gumel AB (2008). Curtailing smoking dynamics: a mathematical modeling approach. Applied Mathematics and Computation, 195(2): 475-499.

Singh J, Kumar D, Al Qurashi M, and Baleanu D (2017). A new fractional model for giving up smoking dynamics. Advances in Difference Equations, 2017(1): 88-102.

Zaman G (2011a). Qualitative behavior of giving up smoking models. Bulletin of the Malaysian Mathematical Sciences Society, 34(2): 403-415.

Zaman G (2011b). Optimal campaign in the smoking dynamics. Hindawi Publishing Corporation, Computational and Mathematical Methods in Medicine, 2011: Article ID 163834, 9 pages. http://dx.doi.org/10.1155/2011/163834

Zeb A, Chohan MI, and Zaman G (2012). The homotopy analysis method for approximating of giving up smoking model in fractional order. Applied Mathematics, 3(8): 914-919.